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HEAVY-NEUTRINO EFFECTS ON τ -LEPTON DECAYS

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ABSTRACT

Minimal extensions of the Standard Model that are motivated by grand unified theories or superstring models with an E_6 symmetry can naturally predict heavy neutrinos of Dirac or Majorana nature. Such heavy neutral leptons violate the decoupling theorem at the one-loop electroweak order and hence offer a unique chance for possible lepton-flavour decays of the τ lepton, *e.g.* $\tau \rightarrow eee$ or $\tau \rightarrow \mu\mu\mu$, to be seen in LEP experiments. We analyze such decays in models with three and four generations.

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Recently, it has been observed that the Standard Model (SM) with more than one right-handed neutrino can dramatically relax [1,2,3] the suppression of heavy-light neutrino mixing $s_L^{\nu_l} (\sim \sqrt{m_{\nu_l}/m_N})$ as derived in usual “see-saw” scenarios [4,5]. High Dirac mass terms are then allowed to be present in the theory without contradicting low-energy constraints on the light neutrino masses. As an immediate phenomenological consequence, it was originally found that the one-loop vertex function relevant for the lepton-flavour-violating decay of the Higgs boson [6] shows a strong quadratic dependence of the heavy neutrino mass. Such nondecoupling effects originating from heavy Majorana neutrino masses have been taken into account in the leptonic flavour-changing decays of the Z boson, leading to rates that could be probed at the CERN Large Electron Positron Collider (LEP) [7]. A similar enhancement due to heavy neutrinos has recently been found to take place in leptonic diagonal Z -boson decays, yielding sizeable non-universality effects [8].

In this note we would like to analyze the phenomenological implications of unified theories for the three-body decays of the τ lepton into other three charged leptons, which we denote hereafter as l , l_1 , and \bar{l}_2 . In fact, we find analytically that the decay amplitude of $\tau \rightarrow ll_1\bar{l}_2$ increase quadratically with the mass of the heavy Dirac or Majorana neutrino, which explicitly violates the decoupling theorem [9]. In particular, we find quantitatively that the decays, $\tau \rightarrow e^-e^-e^+$ and $\tau \rightarrow e^-\mu^-\mu^+$ [or the complementary decays, *i.e.*, $\tau \rightarrow \mu^-\mu^-\mu^+$ and $\tau \rightarrow \mu^-e^-e^+$], deserve the biggest attention from the phenomenological point of view. For completeness, we will present results for the decays $Z \rightarrow e\tau$ or $Z \rightarrow \mu\tau$, using updated constraints for lepton-violating mixing angles. Previous works on flavour-changing decays of the Z boson in a variety of models may be found in Refs. [10,11]

In brief, we first outline the basic low-energy structure of the two most popular extensions of the SM that can naturally account for very light or strictly massless neutrinos. The field content of these models is inspired by heterotic superstring models [12] or certain grand unified theories (GUTs) based on the $SO(10)$ gauge group [13]. The low-energy limit of such theories can be realized in (i) the SM with right-handed neutrinos [4,14,2] and (ii) the SM with left-handed and right-handed neutral singlets [12,13,15]. In addition, we will consider enhancements resulting from a possible fourth sequential family of leptons and quarks. Adopting now the notation of Ref. [2], the Yukawa sector of the SM with a number

n_R of right-handed neutrinos, ν_{Ri}^0 , in addition to n_G left-handed ones, ν_{Li}^0 , reads

$$-\mathcal{L}_Y^\nu = \frac{1}{2}(\bar{\nu}_L^0, \bar{\nu}_R^{0C}) M^\nu \begin{pmatrix} \nu_L^{0C} \\ \nu_R^0 \end{pmatrix} + H.c., \quad (1)$$

where the $(n_G + n_R) \times (n_G + n_R)$ -dimensional neutrino-mass matrix M^ν is given by

$$M^\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_M \end{pmatrix} \quad (2)$$

Since M^ν is a complex symmetric matrix, it can always be diagonalized by an $(n_G + n_R) \times (n_G + n_R)$ unitary matrix U^ν according to the common prescription: $U^{\nu T} M^\nu U^\nu = \hat{M}^\nu$. We identify the first n_G mass eigenstates, ν_i , with the known n_G light neutrinos (*i.e.*, $n_G = 3$), while the remaining n_R mass eigenstates, N_j , are novel heavy Majorana neutrinos predicted by the model. The quark sector of such an extension can completely be described by the SM. The couplings of the charged- and neutral-current interactions are mediated by the mixing matrices B and C , respectively. For more details, the reader is referred to [2]. B and C are correspondingly $n_G \times (n_R + n_G)$ - and $(n_G + n_R) \times (n_G + n_R)$ -dimensional matrices, which are defined as

$$B_{lj} = \sum_{k=1}^{n_G} V_{lk} U_{kj}^{\nu*} \quad \text{and} \quad C_{ij} = \sum_{k=1}^{n_G} U_{ki}^\nu U_{kj}^{\nu*}, \quad (3)$$

where V is the leptonic Cabbibo-Kobayashi-Maskawa (CKM) matrix. Note that the flavour-mixing matrices B and C satisfy a number of identities that have been forced by the renormalizability of the model [2]. With the help of these identities, one can derive useful relations between mixings B , C and heavy neutrino masses. For a model with two-right handed neutrinos, for example, we obtain

$$B_{lN_1} = \frac{\rho^{1/4} s_L^{\nu_l}}{\sqrt{1 + \rho^{1/2}}}, \quad B_{lN_2} = \frac{is_L^{\nu_l}}{\sqrt{1 + \rho^{1/2}}}, \quad (4)$$

where $\rho = m_{N_2}^2/m_{N_1}^2$ is a mass ratio of the two heavy Majorana neutrinos N_1 and N_2 that are predicted in such a model, and $s_L^{\nu_l}$ is defined as [16]

$$(s_L^{\nu_l})^2 \equiv \sum_{j=1}^{n_R} |B_{lN_j}|^2 \simeq \left(m_D^\dagger \frac{1}{m_M^2} m_D \right)_{ll}. \quad (5)$$

Furthermore, the mixings $C_{N_i N_j}$ can easily be obtained by

$$\begin{aligned} C_{N_1 N_1} &= \frac{\rho^{1/2}}{1 + \rho^{1/2}} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2, & C_{N_2 N_2} &= \frac{1}{1 + \rho^{1/2}} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2, \\ C_{N_1 N_2} &= -C_{N_2 N_1} = \frac{i\rho^{1/4}}{1 + \rho^{1/2}} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2. \end{aligned} \quad (6)$$

Our minimal scenario will then depend only on the masses of the heavy Majorana neutrinos, m_{N_1} and m_{N_2} [or equivalently on m_{N_1} and ρ], and the mixing angles $(s_L^{\nu_i})^2$, which are directly constrained by low-energy data.

Another attractive scenario can be considered a superstring-inspired extension of the SM, in which left-handed neutral singlets, S_{Li} , in addition to the right-handed neutrinos, ν_{Ri}^0 , have been introduced. In this scenario, the light neutrinos are strictly massless to all orders of perturbation theory, if $\Delta L = 2$ interactions are absent from the model [13]. For the sake of simplicity, we will assume that the number of right-handed neutrinos, n_R , equals the number of the singlet fields S_{Li} . After the spontaneous break-down of the SM gauge symmetry, the Yukawa sector relevant for the neutrino masses is given by [12,13]

$$-\mathcal{L}_Y^\nu = \frac{1}{2}(\bar{\nu}_L^0, \bar{\nu}_R^{0C}, \bar{S}_L) \mathcal{M}^\nu \begin{pmatrix} \nu_L^{0C} \\ \nu_R^0 \\ S_L^C \end{pmatrix} + H.c., \quad (7)$$

where the $(n_G + 2n_R) \times (n_G + 2n_R)$ neutrino-mass matrix takes the form

$$\mathcal{M}^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & 0 \end{pmatrix}. \quad (8)$$

Since the neutrino matrix in Eq. (8) has rank $2n_R$, this implies that n_G eigenvalues of \mathcal{M}^ν will be zero. These n_G massless eigenstates are identified with the ordinary light neutrinos, ν_e , ν_μ and ν_τ [12,13]. The remaining $2n_R$ Weyl fermions are degenerate in pairs due to the fact that L is conserved and so form n_R heavy Dirac neutrinos. A nice feature of the model is that the individual leptonic quantum numbers may be violated [15,20]. The charged-current and neutral-current interactions of the SM with left-handed and right-handed isosinglets can be found in Ref. [15]. To a good approximation, we assume that possible novel particles related to the above unified theories, such as Pati-Salam leptoquarks or extra charged and

neutral gauge bosons are sufficiently heavy so as to decouple completely from our low-energy processes.

Unified theories are constrained by a number of low-energy experiments [16] and LEP data [17]. Experimental tests giving stringent constraints turn out to be the neutrino counting at the Z peak, the precise measurement of the muon width $\mu \rightarrow e\nu_e\nu_\mu$, charged-current universality effects on $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$, non-universality effects on $B(\tau \rightarrow e\nu\nu)/B(\tau \rightarrow \mu\nu\nu)$, etc. All these constraints, which are derived by the low-energy data mentioned above, depend, more or less, on the gauge structure of the model under consideration. In particular, interesting phenomenology could arise from possible decays $Z \rightarrow N^*\nu$ at LEP, in case $m_N \lesssim M_Z$ [18]. For the present analysis, we consider that all heavy neutrinos are much heavier than M_Z , and thus tolerating the following upper limits [17]:

$$(s_L^{\nu_e})^2, (s_L^{\nu_\mu})^2 < 0.015, \quad (s_L^{\nu_\tau})^2 < 0.070, \quad \text{and} \quad (s_L^{\nu_\mu})^2 (s_L^{\nu_e})^2 < 1 \cdot 10^{-8}. \quad (9)$$

The last constraint in Eq. (9) comes from the non-observation of the decay mode $\mu \rightarrow e\gamma$ or $\mu \rightarrow eee$. Another limitation to the parameters of our model comes from the requirement of the validity of perturbative unitarity that can be violated in the limit of large heavy-neutrino masses. A qualitative estimate for the latter may be obtained by requiring that the total widths, Γ_{N_i} , and masses of neutrino fields N_i satisfy the inequality $\Gamma_{N_i}/m_{N_i} < 1/2$. In the limit of $m_{N_i} \gg M_W, M_Z, M_H$, the afore-mentioned requirement leads to [2]

$$\frac{\alpha_w}{4M_W^2} m_{N_i}^2 |C_{N_i N_i}|^2 < 1/2, \quad (10)$$

with $\alpha_w = g_w^2/4\pi$.

Since Eq. (9) tells us that either $(s_L^{\nu_e})^2$ or $(s_L^{\nu_\mu})^2$ and not both of them could be as large as 0.01, we will assume, for example, that $(s_L^{\nu_\mu})^2 \simeq 0$. Consequently, $B(\tau^- \rightarrow e^- e^- \mu^+)$, $B(\tau^- \rightarrow \mu^- \mu^- e^+)$ will be vanishingly small. There are then two possible decays that are of potential interest, *i.e.*,

$$\begin{aligned} \text{a.} \quad & \tau^- \rightarrow e^- \mu^- \mu^+, \\ \text{b.} \quad & \tau^- \rightarrow e^- e^- e^+. \end{aligned} \quad (11)$$

Of course, one would equally assume that $(s_L^{\nu_e})^2 \simeq 0$ and $(s_L^{\nu_\mu})^2 \simeq 0.01$. In such a case, the complementary decays where e is replaced by muon in Eq. (11) and vice versa will

be of interest. Furthermore, we have to stress the fact that a simultaneous observation of $\tau \rightarrow eee$ and $\tau \rightarrow \mu\mu\mu$ cannot be compatible with experiments leading to the third inequality of Eq. (9). The matrix element relevant for the decay $\tau(p_\tau) \rightarrow l(p_l)l_1(p_1)\bar{l}_2(p_2)$ gets contributions from γ - and Z -mediated graphs that may be found in Ref. [7] and box diagrams shown in Fig. 1. These three transition elements are generically written down as follows:

$$\begin{aligned} \mathcal{T}_\gamma(\tau \rightarrow ll_1\bar{l}_2) &= -\frac{i\alpha_w^2 s_w^2}{4M_W^2} \delta_{l_1 l_2} \bar{u}_{l_1} \gamma^\mu v_{l_2} \bar{u}_l \left[F_\gamma^{\tau l} (\gamma_\mu - \frac{q_\mu \not{q}}{q^2}) (1 - \gamma_5) \right. \\ &\quad \left. - i G_\gamma^{\tau l} \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_\tau (1 + \gamma_5) + m_l (1 - \gamma_5)) \right] u_\tau, \end{aligned} \quad (12)$$

$$\mathcal{T}_Z(\tau \rightarrow ll_1\bar{l}_2) = -\frac{i\alpha_w^2}{16M_W^2} \delta_{l_1 l_2} F_Z^{\tau l} \bar{u}_l \gamma_\mu (1 - \gamma_5) u_\tau \bar{u}_{l_1} \gamma^\mu (1 - 4s_w^2 - \gamma_5) v_{l_2} \quad (13)$$

$$\mathcal{T}_{Box}(\tau \rightarrow ll_1\bar{l}_2) = -\frac{i\alpha_w^2}{16M_W^2} F_{Box}^{\tau ll_1 l_2} \bar{u}_l \gamma_\mu (1 - \gamma_5) u_\tau \bar{u}_{l_1} \gamma^\mu (1 - \gamma_5) v_{l_2}, \quad (14)$$

where $q = p_1 + p_2$, $s_w^2 = 1 - M_W^2/M_Z^2$, and with $\lambda_i = m_i^2/M_W^2$ (summation over light and heavy Majorana states with masses m_i implied),

$$F_\gamma^{\tau l} = \sum_i B_{\tau i}^* B_{li} F_\gamma(\lambda_i), \quad (15)$$

$$G_\gamma^{\tau l} = \sum_i B_{\tau i}^* B_{li} G_\gamma(\lambda_i), \quad (16)$$

$$F_Z^{\tau l} = \sum_{ij} B_{\tau i}^* B_{lj} \left[\delta_{ij} F_Z(\lambda_i) + C_{ij}^* G_Z(\lambda_i, \lambda_j) + C_{ij} H_Z(\lambda_i, \lambda_j) \right], \quad (17)$$

$$\begin{aligned} F_{Box}^{\tau ll_1 l_2} &= \sum_{ij} B_{\tau i}^* B_{l_2 j}^* (B_{li} B_{l_1 j} + B_{l_1 i} B_{lj}) F_{Box}(\lambda_i, \lambda_j) \\ &\quad + \sum_{ij} B_{\tau i}^* B_{l_2 i}^* B_{lj} B_{l_1 j} G_{Box}(\lambda_i, \lambda_j), \end{aligned} \quad (18)$$

are composite form factors that include multiplicative factors of certain combinations of B and C matrices. The photonic Inami-Lim form factors F_γ and G_γ [21], as well as the form factors F_Z [21], H_Z , G_Z , F_{Box} , and G_{Box} are to be presented analytically in Ref. [22]. It is, however, useful to discuss the asymptotic limit of the composite form factors described above in a model with two heavy Majorana neutrinos. Using the expressions of Eqs. (4) and (6) for the mixing matrices B and C , we find that for $\lambda_{N_1} = m_{N_1}^2/M_W^2 \gg 1$ and $\rho = m_{N_2}^2/m_{N_1}^2 \gg 1$,

$$F_\gamma^{\tau l} \rightarrow -\frac{1}{6} s_L^{\nu_\tau} s_L^{\nu_l} \ln \lambda_{N_1}, \quad (19)$$

$$G_\gamma^{\tau l} \rightarrow \frac{1}{2} s_L^{\nu_\tau} s_L^{\nu_l}, \quad (20)$$

$$F_Z^{\tau l} \rightarrow -\frac{3}{2} s_L^{\nu_\tau} s_L^{\nu_l} \ln \lambda_{N_1} + s_L^{\nu_\tau} s_L^{\nu_l} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2 \frac{\lambda_{N_1}}{(1 + \rho^{\frac{1}{2}})^2} \left(-\frac{3}{2} \rho + \frac{-\rho + 4\rho^{\frac{3}{2}} - \rho^2}{4(1 - \rho)} \ln \rho \right), \quad (21)$$

$$F_{Box}^{\tau ll_1 l_2} \rightarrow -(s_L^{\nu_\tau} s_L^{\nu_l} \delta_{l_1 l_2} + s_L^{\nu_\tau} s_L^{\nu_{l_1}} \delta_{ll_2}) + s_L^{\nu_\tau} s_L^{\nu_l} s_L^{\nu_{l_1}} s_L^{\nu_{l_2}} \frac{\lambda_{N_1}}{(1 + \rho^{\frac{1}{2}})^2} \left(-\rho - \frac{\rho + \rho^{\frac{3}{2}} + \rho^2}{1 - \rho} \ln \rho \right). \quad (22)$$

In the limit $\rho \rightarrow 1$ and for $\lambda_{N_1} \sim \lambda_{N_2} \sim \lambda_N \gg 1$, Eqs. (21) and (22) take the form

$$F_Z^{\tau l} \rightarrow -\frac{3}{2} s_L^{\nu_\tau} s_L^{\nu_l} \ln \lambda_N - \frac{1}{2} s_L^{\nu_\tau} s_L^{\nu_l} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2 \lambda_N, \quad (23)$$

$$F_{Box}^{\tau ll_1 l_2} \rightarrow -(s_L^{\nu_\tau} s_L^{\nu_l} \delta_{l_1 l_2} + s_L^{\nu_\tau} s_L^{\nu_{l_1}} \delta_{ll_2}) + \frac{1}{2} s_L^{\nu_\tau} s_L^{\nu_l} s_L^{\nu_{l_1}} s_L^{\nu_{l_2}} \lambda_N. \quad (24)$$

From Eqs. (19)–(24), it is then obvious that all the one-loop functions, $F_\gamma^{\tau l}$, $G_\gamma^{\tau l}$, $F_Z^{\tau l}$, and $F_{Box}^{\tau ll_1 l_2}$, violate the decoupling theorem [9]. Such a violation is a common feature for all theories based on the spontaneous symmetry breaking mechanism. Taking the dominant nondecoupling parts of the composite form factors into account, we arrive at the simple expression for the branching ratios:

$$\begin{aligned} B(\tau^- \rightarrow e^- \mu^- \mu^+) &\simeq \frac{\alpha_w^4}{24576\pi^3} \frac{m_\tau^4}{M_W^4} \frac{m_\tau}{\Gamma_\tau} \left[|F_{Box}^{\tau e \mu \mu}|^2 + 2(1 - 2s_w^2) \text{Re}[F_Z^{\tau e} F_{Box}^{\tau e \mu \mu*}] \right. \\ &\quad \left. + 8s_w^4 |F_Z^{\tau e}|^2 \right] \\ &\simeq \frac{\alpha_w^4}{98304\pi^3} \frac{m_\tau^4}{M_W^4} \frac{m_\tau}{\Gamma_\tau} \frac{m_N^4}{M_W^4} (s_L^{\nu_\tau})^2 (s_L^{\nu_e})^2 \left\{ (s_L^{\nu_\mu})^4 \right. \\ &\quad \left. + 2(1 - 2s_w^2) (s_L^{\nu_\mu})^2 \sum_i (s_L^{\nu_i})^2 + 8s_w^4 \left[\sum_i (s_L^{\nu_i})^2 \right]^2 \right\}. \end{aligned} \quad (25)$$

In the same heavy neutrino limit, we obtain

$$\begin{aligned} B(\tau^- \rightarrow e^- e^- e^+) &\simeq \frac{\alpha_w^4}{24576\pi^3} \frac{m_\tau^4}{M_W^4} \frac{m_\tau}{\Gamma_\tau} \left[\frac{1}{2} |F_{Box}^{\tau e e e}|^2 + 2(1 - 2s_w^2) \text{Re}[F_Z^{\tau e} F_{Box}^{\tau e e e*}] \right. \\ &\quad \left. + 12s_w^4 |F_Z^{\tau e}|^2 \right] \\ &\simeq \frac{\alpha_w^4}{98304\pi^3} \frac{m_\tau^4}{M_W^4} \frac{m_\tau}{\Gamma_\tau} \frac{m_N^4}{M_W^4} (s_L^{\nu_\tau})^2 (s_L^{\nu_e})^2 \left\{ \frac{1}{2} (s_L^{\nu_e})^4 \right. \\ &\quad \left. + 2(1 - 2s_w^2) (s_L^{\nu_e})^2 \sum_i (s_L^{\nu_i})^2 + 12s_w^4 \left[\sum_i (s_L^{\nu_i})^2 \right]^2 \right\}. \end{aligned} \quad (26)$$

In Eqs. (25) and (26), Γ_τ denotes the total decay width of the τ lepton, which is experimentally measured to be $\Gamma_\tau = 2.16 \cdot 10^{-12}$ GeV [19].

Apart from the τ -lepton decays given in Eq. (11), the decay $Z \rightarrow e\tau$ can also be enhanced due to the same heavy neutrino effects to an extent that may be seen at LEP [7]. To the leading order of heavy neutrino masses ($m_N \gg M_W$), the branching ratio of this decay mode is obtained by

$$\begin{aligned} B(Z \rightarrow \tau^- e^+ + e^- \tau^+) &= \frac{\alpha_w^3}{48\pi^2 c_w^3} \frac{M_W}{\Gamma_Z} |\mathcal{F}_Z^{e\tau}(M_Z^2)|^2 \\ &\simeq \frac{\alpha_w^3}{768\pi^2 c_w^3} \frac{M_W}{\Gamma_Z} \frac{m_N^4}{M_W^4} (s_L^{\nu_e})^2 (s_L^{\nu_\tau})^2 \left[\sum_i (s_L^{\nu_i})^2 \right]^2, \end{aligned} \quad (27)$$

where Γ_Z is the total width of the Z boson. Note that $\mathcal{F}_Z^{e\tau}(0) = F_Z^{e\tau}/2$.

In order to minimize the free parameters of the theory that could vary independently, we will assume an extension of the SM by two right-handed neutrinos. The neutrino mass spectrum of such a model consists of three light Majorana neutrinos which have been identified with the three known neutrinos, ν_e , ν_μ , and ν_τ , and two heavy ones denoted by N_1 and N_2 . On the other hand, the SM inspired by superstring theories with an E_6 symmetry [12], in which one left-handed and one right-handed chiral singlets are present, can effectively be recovered by the SM with two right-handed neutrinos when going to the degenerate mass limit for the two heavy Majorana neutrinos.

Assuming the maximally allowed values [17] for $(s_L^{\nu_\tau})^2 = 0.07$ and $(s_L^{\nu_e})^2 = 0.015$ ($(s_L^{\nu_\mu})^2 \simeq 0$) given in Eq. (9), we find the encouraging branching ratios

$$B(\tau^- \rightarrow e^- e^- e^+) \lesssim 2 \cdot 10^{-6} \quad \text{and} \quad B(\tau^- \rightarrow e^- \mu^- \mu^+) \lesssim 1 \cdot 10^{-6}, \quad (28)$$

where the upper bounds is estimated by using $m_N \simeq 3$ TeV as derived from Eq. (10). The present experimental upper limits on these decays are [19]

$$B(\tau^- \rightarrow e^- e^- e^+), B(\tau^- \rightarrow e^- \mu^- \mu^+) < 1.4 \cdot 10^{-5}, \quad \text{CL} = 90\%. \quad (29)$$

Even if we assume smaller values for the mixing angles, $(s_L^{\nu_\tau})^2 = 0.035$ and $(s_L^{\nu_e})^2 = 0.01$ ($(s_L^{\nu_\mu})^2 = 0$), the lepton-flavour-violating decays of the τ lepton can still be significant, *i.e.*,

$$B(\tau^- \rightarrow e^- e^- e^+) \lesssim 5 \cdot 10^{-7} \quad \text{and} \quad B(\tau^- \rightarrow e^- \mu^- \mu^+) \lesssim 3 \cdot 10^{-7}. \quad (30)$$

Since the branching ratio increase with the heavy neutrino mass to the fourth power, this strong mass dependence gives rise to measurable values for the leptonic three-body decays of the τ lepton. To be precise, if we had neglected contributions of seemingly suppressed terms $\sim (s_L^{\nu_i})^4$ in the transition amplitude, we would then have found a reduction of our numerical values up to $\sim 10^{-2}$. In the low-mass range of heavy neutrinos (*i.e.* for $m_{N_i} < 200$ GeV) the difference between the complete and the approximate computation is quite small and consistent with results obtained in [20]. For very heavy neutrinos, the situation is quite different, since in the decay amplitude, terms proportional to $(s_L^{\nu_i})^2$ increase logarithmically with the heavy neutrino mass m_N , *i.e.* $\ln(m_N^2/M_W^2)$, while terms of $\mathcal{O}((s_L^{\nu_i})^4)$ show a strong quadratic dependence in the heavy neutrino mass, *i.e.* m_N^2/M_W^2 . Finally, τ leptons can also decay hadronically via the channels: $\tau \rightarrow l_i \eta$, $\tau \rightarrow l_i \pi^0$, etc. [20]. Since present experimental sensitivity to these decays is rather weak [19], *e.g.*, $B(\tau \rightarrow e \pi^0) < 1.4 \cdot 10^{-4}$, at CL= 90%, one could expect that it would be difficult to probe heavy neutrino effects in such hadronic decay channels.

We will now investigate the LEP potential of observing lepton-flavour-violating decays at the Z peak. Since we always assume that $(s_L^{\nu_\mu})^2 \simeq 0$ for reasons mentioned above, we will focus our analysis on the decays $Z \rightarrow e^- \tau^+$ or $e^+ \tau^-$. Within the perturbatively allowed range of heavy neutrino masses, we find

$$\begin{aligned} B(Z \rightarrow e^- \tau^+ + e^+ \tau^-) &\lesssim 4.0 \cdot 10^{-6}, \quad \text{for } (s_L^{\nu_\tau})^2 = 0.070, \quad (s_L^{\nu_e})^2 = 0.015, \\ B(Z \rightarrow e^- \tau^+ + e^+ \tau^-) &\lesssim 1.1 \cdot 10^{-6}, \quad \text{for } (s_L^{\nu_\tau})^2 = 0.035, \quad (s_L^{\nu_e})^2 = 0.010, \\ B(Z \rightarrow e^- \tau^+ + e^+ \tau^-) &\lesssim 6.0 \cdot 10^{-7}, \quad \text{for } (s_L^{\nu_\tau})^2 = 0.020, \quad (s_L^{\nu_e})^2 = 0.010. \end{aligned} \quad (31)$$

All these branching ratios could be detected at future LEP data, as the present experimental sensitivity at LEP is [19]

$$B(Z \rightarrow e^- \tau^+ + e^+ \tau^-) < 1.3 \cdot 10^{-5}, \quad \text{CL} = 95\%. \quad (32)$$

However, the present upper bound on the flavour-changing Z -boson decays do not yet impose any severe constraints on our analysis of τ -lepton decays.

In the following, we will briefly discuss the phenomenological consequences induced by the presence of an extra sequential family. Such scenarios have recently received much attention due to the additional fact that they could naturally resuscitate extended technicolour theories [23]. LEP precision experiments provide a useful framework to either

constrain or establish such extended models, when one analyzes electroweak oblique parameters [24,25,26] and other quantum effects [27]. As a consequence of such an analysis, the lightest of the two heavy Majorana neutrinos belonging to the fourth family cannot be heavier than 1 TeV. In our models, the inclusion of an extra family amounts to replacing $\sum_i (s_L^{\nu_i})^2 \rightarrow 1$ in Eqs. (25), (26), and (27). The branching-ratio values we obtain in such models are larger, *i.e.*,

$$B(\tau^- \rightarrow e^- e^- e^+), B(\tau^- \rightarrow e^- \mu^- \mu^+) \lesssim 4 \cdot 10^{-6}, \quad (33)$$

$$B(Z \rightarrow e^- \tau^+ + e^+ \tau^-) \lesssim 8 \cdot 10^{-6}. \quad (34)$$

In conclusion, we have explicitly demonstrated that GUT or superstring-inspired extensions of the minimal SM can naturally account for sizeable branching ratios of the τ -lepton decays of the type $\tau \rightarrow eee$, $\tau \rightarrow \mu\mu\mu$, etc. These decays show a strong *quadratic* mass dependence of the heavy neutrino mass (see Eqs. (25) and (26)), which gives a unique chance for such non-SM signals to be seen in present or future τ factories.

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Figure Caption

Fig. 1: Feynman diagrams relevant for the leptonic decays $\tau \rightarrow ll_1\bar{l}_2$.

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